Minimum-Weight Design of Stiffened Cylinders under Axial Compression

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A procedure is outlined by which one may design a stiffened circular cylindrical shell under a given uniform axial compression with minimum weight. By judicious choice of the objective function and proper grouping of the parameters involved, the solution is accomplished by separation into 2 phases: the "optimization" phase and the "design" phase. The "optimization" phase yields design charts, which are then used in the "design" phase to arrive at the minimum weight configuration. A critical review of previous investigations and a comparison with their resulting configurations are presented. The stiffener geometry used in this comparison is rectangular, because it represents the only configuration which has been dealt with to some degree of success in the past.

Nomenclature

A_x = stringer cross-sectional area
A_y = ring cross-sectional area
D = flexural stiffness of the skin
E = skin Young's modulus of elasticity
E_x = stringer Young's modulus of elasticity
E_y = ring Young's modulus of elasticity
GB = ratio of applied load to general instability load, $\bar{N}/\bar{N}_{xx_{cr}}$
I_{xc} = stringer moment of inertia about its centroidal axes
I_{yc} = ring moment of inertia about its centroidal axes
$\bar{K}_{xx}$ = axial compressive buckling load coefficient
L = total length of the shell
MG = minimum gage
$\bar{N}$ = applied uniform axial compressive load
$\bar{N}_{xx_{cr}}$ = critical uniform axial compressive load
$\bar{N}^*$ = nondimensional load parameter
PB = ratio of applied load to panel buckling load $\bar{N}/\bar{N}_{xxp_{cr}}$
R = radius of the shell
$RSRR$ = rectangular stringer and rectangular ring
RYT = ring yielding coefficient in tension, σ_{yyr}/σ_{yr}
SB = skin buckling coefficient, $\sigma_{xxsk}/\sigma_{xxsk_{cr}}$
STB = stringer buckling coefficient, $\sigma_{xxst}/\sigma_{xxst_{cr}}$
$STYC$ = stringer yielding coefficient in compression, $\sigma_{xxst}/\sigma_{yst}$
SY = skin yielding coefficient, $\sigma_{xxsk}/\sigma_{ysk}$
W = weight of the shell
$\bar{W}$ = nondimensional weight parameter
W^* = composite weight function
$\bar{W}^*$ = nondimensional composite weight function
WMG = without minimum gage
Z = curvature parameter, $L^2(1-v^2)^{1/2}/Rh$
d = stiffener depth
e_x = stringer eccentricity
e_y = ring eccentricity
$\bar{e}_x$ = nondimensional stringer eccentricity
$\bar{e}_y$ = nondimensional ring eccentricity
h = skin thickness
l_x = stringer spacing
l_y = ring spacing
m = number of axial waves for general instability
n = number of circumferential waves for general instability
m_p = number of axial waves for panel buckling
n_p = number of circumferential waves for panel buckling
t_x = stringer thickness
t_y = ring thickness
u, v, w = displacement in the x, y, z directions
$\bar{\alpha}_x$ = nondimensional stringer radius of gyration
$\bar{\alpha}_y$ = nondimensional ring radius of gyration
λ = Lagrange multiplier
$\bar{\lambda}_{xx}$ = nondimensional stringer extensional stiffness parameter
$\bar{\lambda}_{yy}$ = nondimensional ring extensional stiffness parameter
ρ_{sk} = weight density of the skin
ρ_x = weight density of the stringer
ρ_y = weight density of the ring
$\bar{\rho}_{xx}$ = nondimensional stringer flexural stiffness parameter
$\bar{\rho}_{yy}$ = nondimensional ring flexural stiffness parameter
v = Poisson's ratio
σ_{cr} = buckling stress
σ_{xxsk} = prebuckling stress of the skin in the x-direction
σ_{yysk} = prebuckling stress of the skin in the y-direction
σ_{xxst} = prebuckling stress of the stringer
σ_{yyr} = prebuckling stress of the ring
σ_{yr} = yield stress of ring material
σ_{ysk} = yield stress of skin material
σ_{yst} = yield stress of stringer material

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I. Introduction

As the size of modern aerospace vehicles increases, the demand for light weight structures increases. This has made the structural engineer, engaging in this area, more and more conscious of minimum weight design. A structural configuration that is used widely in aerospace vehicles is the stiffened thin cylindrical shell. Since stiffened cylindrical shells have been used extensively in the past 30 yr, a tremendous effort has been exerted in designing such a configuration for minimum weight. Gerard[1] has presented a comprehensive bibliography on the subject of optimal structural design. His work has been extended by Niordson and Pedersen.[2] Better understanding, during the past decade, of the failure modes of the stiffened thin cylindrical shell for aerospace use has produced some important results in the attempt to achieve minimum-weight design.[3–16] A detailed discussion of these efforts is presented in the Sec. II.

The precise statement of the problem considered in this paper is as follows: given an internally stiffened circular cylindrical shell of specified material, radius and length, find the size and spacings of the rectangular stiffeners and the thickness of the skin such that it can safely carry a given uniform axial compression with minimum weight. The design objective is minimum weight. The general instability load is taken to be the equality constraint, because it represents the principal catastrophic mode of failure. Panel instability which is also a catastrophic mode of failure is considered as an inequality constraint, which means that the material is distributed in such a way that this mode of failure is avoided. Other behavioral inequality constraints are wrinkling of the skin, local instability of the stringers, and limitations on the stress level of the skin, stringers, and rings. In addition, geometric inequality constraints are used, which represent realistic dimensions for the design variables (thickness and height of stiffeners, spacings etc.).

The solution to this problem is accomplished in 2 stages. First, by a proper grouping of the design variables, the number of parameters that optimizes the weight is minimized. On the basis of this a mathematical search technique is employed, and design charts are prepared which clearly show the effect of these few parameters on the weight of the shell. This first stage is called the "optimization" phase. Next, these charts are employed to arrive at the minimum weight configuration satisfying all constraints. This stage is called the "design" phase.

This procedure, effectively, leads to a minimum weight configuration against general instability and satisfies all other possible constraints (behavioral as well as geometric). Finally, although the stiffeners are taken to be rectangular the procedure can easily be extended to find the best shape of the stiffening geometry.

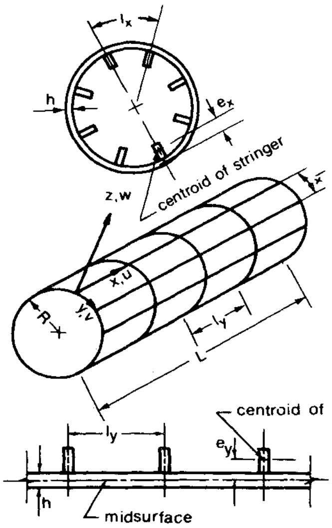

Fig. 1 Shell geometry.

II. Review of Previous Work

In the past, there have been 2 types of attempt at the minimum weight design of the thin circular cylindrical shell subject to a uniform axial compression. One approach is to make a parametric study with regard to the general instability mode of failure and investigate the effects of various parameters on the cylinder weight,[3-5,7-10] by keeping several paramters fixed. These investigations are also based on the premise that the minimum weight is accomplished if all possible modes of failure occur simultaneously. This conjecture is disproved by the results of another group of investigators[11-16] who do not impose this limitation on their formulations. In addition, recently Thompson and Lewis[20] have qualitatively verified the suspicion of van der Neut,[17] Koiter and Kuiken,[18] and Graves-Smith,[19] that a structural element which is designed for simultaneous occurrence of all possible modes of failure is extremely sensitive to geometric imperfections. Because of these two reasons the resulting designs based on this approach are somewhat questionable. The second approach is based on convenient mathematical search techniques, and the objective function contains all of the constraints as penalty functions. These penalty functions as well as the initial objective function (weight) are expressed in terms of the design variables. This approach, used in Refs. 11–16, is in accord with the philosophy of the present time, which is to achieve a fully automated design, but the authors have serious reservations as to the desirability and the useful applicability of such techniques. The number of the design variables for rectangular cross-sectional stiffeners is 7. Admittedly, all of the investigators who have used mathematical search techniques in the 7-dimensional space have reported great difficulties and computational failures. Moreover, if one were to deal with T-shaped stiffeners, the number of design variables would be 11 and hence more computational difficulties would arise. Even if these difficulties can be overcome, the authors still question the applicability of such approaches for a number of reasons. First, because of the complete automation the designer is virtually divorced from the design procedure and his control in the presence of new information available, in the presence of changing constraints, and in the presence of shifting emphasis as far as the objective is concerned. Second, due to various behavioral constraints built into their objective functions, their designs cannot purposely avoid the simultaneous occurrence of the various instability failure modes. Thus, the resulting minimum weight configuration may be unnecessarily more imperfection sensitive.

III. Mathematical Formulation of the Problem

Optimization Phase

Assuming that the eccentricities of the stiffeners are small in comparison with the radius of the stiffened cylindrical shell such that the common stiffener material at the intersection of stringers and rings is negligible, then the weight of the stiffened cylindrical shell is given by (see Fig. 1)

$$W = 2\pi RLh\rho_{sk} + \rho_x \int_0^L \int_0^{2\pi R} \frac{A_x}{l_x}\, dy\, dx + \rho_y \int_0^L \int_0^{2\pi R} \frac{A_y}{l_y}\, dy\, dx \qquad (1)$$

Define the following nondimensional parameters.

$$\bar{\lambda}_{xx} = \frac{E_x A_x(1-\nu^2)}{Ehl_x};\quad \bar{\lambda}_{yy} = \frac{E_y A_y(1-\nu^2)}{Ehl_y};\quad \bar{\rho}_{xx} = \frac{E_x I_{xc}}{Dl_x};$$

$$\bar{\rho}_{yy} = \frac{E_y I_{yc}}{Dl_y};\quad \bar{e}_x = \frac{\pi^2 Re_x}{L^2};\quad \bar{e}_y = \frac{\pi^2 Re_y}{L^2};\quad Z = \frac{L^2(1-\nu^2)^{1/2}}{Rh};$$

$$\bar{\beta} = \frac{nL}{m\pi R} = \frac{\beta}{m};\quad K_{xx} = \frac{\bar{N}L^2}{\pi^2 D}$$

Using these new parameters, it can be shown from Ref. 21 that the classical general instability buckling parameter of the thin stiffened cylindrical shell subject to a uniform axial compression with simply-supported boundary conditions is

$$\bar{K}_{xx} = Pm^2 + \frac{Q}{m^2} + S \qquad (2)$$

where

$$P = 1+\rho_{xx}+2\bar{\beta}^2+(1+\bar{\rho}_{yy})\bar{\beta}^4 + \frac{12Z^2}{\pi^4(1-\nu^2)}\Big[\bar{e}_x^{\,2}\bar{\lambda}_{xx} + \frac{2}{1-\nu}\bar{e}_x^{\,2}\bar{\lambda}_{xx}(1-\nu+\bar{\lambda}_{yy})\bar{\beta}^2 + \Big\{\bar{e}_x^{\,2}\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \frac{2(1+\nu)}{(1-\nu)}\bar{\lambda}_{xx}\bar{\lambda}_{yy}\bar{e}_x\bar{e}_y + \bar{e}_y^{\,2}\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\Big\}\bar{\beta}^4 + \frac{2}{1-\nu}\bar{e}_y^{\,2}\bar{\lambda}_{yy}(1-\nu+\bar{\lambda}_{xx})\bar{\beta}^6 + \bar{e}_y^{\,2}\bar{\lambda}_{yy}\bar{\beta}^8\Big] \Big/ \Big[1+\bar{\lambda}_{xx} + \frac{2}{1-\nu}\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu\}\bar{\beta}^2 + (1+\bar{\lambda}_{yy})\bar{\beta}^4\Big]$$

$$Q = \frac{12Z^2}{\pi^4(1-\nu^2)}\left[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2\right] \Big/ \Big[1+\bar{\lambda}_{xx} + \frac{2}{1-\nu}\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu\}\bar{\beta}^2 + (1+\bar{\lambda}_{yy})\bar{\beta}^4\Big]$$

$$S = \frac{24Z^2}{\pi^4(1-\nu^2)}\Big[\nu\bar{e}_x\bar{\lambda}_{xx} - \{\bar{e}_x\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \bar{e}_y\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\}\bar{\beta}^2 + \nu\bar{e}_y\bar{\lambda}_{yy}\bar{\beta}^4\Big] \Big/ \Big[1+\bar{\lambda}_{xx}\frac{2}{1-\nu} \times \{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu\}\bar{\beta}^2 + (1+\bar{\lambda}_{yy})\bar{\beta}^4\Big]$$

For any given geometry of the stiffened shell under a uniform axial compression, the buckling parameter $\bar{K}_{xx_{cr}}$, is obtained through the minimization of Eq. (2) with respect to all integer values of m and n, except $m = 0$ and $n = 1$.

The panel instability buckling parameter, $\bar{K}_{xxp}$, is obtained from Eq. (2) by setting the nondimensional ring parameters to zero. That is

$$\bar{\lambda}_{yy} = 0, \quad \bar{\rho}_{yy} = 0, \quad \bar{e}_y = 0, \quad \text{and } L = l_y$$

From Eq. (1), the weight of the stiffened shell is

$$W = 2\pi RLh\rho_{sk}\left[1 + \frac{1}{1-\nu^2}\left(\frac{E\rho_x}{E_x\rho_{sk}}\bar{\lambda}_{xx} + \frac{E\rho_y}{E_y\rho_{sk}}\bar{\lambda}_{yy}\right)\right] \tag{3}$$

At this point it is convenient to introduce 2 new parameters $\bar{\alpha}_x$ and $\bar{\alpha}_y$. These new parameters $\bar{\alpha}$ denote the ratio of the radius of gyration of the stiffeners to the radius of gyration of the skin of unit width.

$$\bar{\alpha}_x = (d_x/h); \quad \bar{\alpha}_y(d_y/h)$$

With these new parameters one may easily eliminate the parameters $\bar{e}_x$, $\bar{e}_y$, $\bar{\rho}_{xx}$ and $\bar{\rho}_{yy}$ from Eq. (2), through the following equations

$$\bar{\rho}_{xx} = \bar{\alpha}_x{}^2\bar{\lambda}_{xx}; \quad \bar{\rho}_{yy} = \alpha_y{}^2\bar{\lambda}_{yy}$$

$$\bar{e}_x = \frac{\pi^2(1-\nu^2)^{1/2}}{2Z}(1+\bar{\alpha}_x); \quad \bar{e}_y = \frac{\pi^2(1-\nu^2)^{1/2}}{2Z}(1+\bar{\alpha}_y)$$

Now, one can see that the buckling parameter $\bar{K}_{xx}$, is a function of Z, $\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$, $\bar{\alpha}_x$, $\bar{\alpha}_y$, m^2 and $\bar{\beta}^2$.

$$\bar{K}_{xx} = \bar{K}_{xx}[Z, \bar{\lambda}_{xx}, \bar{\lambda}_{yy}, m^2, \bar{\beta}^2, (\bar{\alpha}_x, \bar{\alpha}_y)] \tag{4}$$

The requirement of minimum weight against general instability leads to the objective function

$$W^* = W + \lambda|\bar{N}_{xx_{cr}} - \bar{N}| \tag{5}$$

where W is the weight of the stiffened shell, $\bar{N}_{xx_{cr}}$ the general instability load, $\bar{N}$ the applied load, and λ a Lagrange multiplier.

Equation (5) can be rearranged so that

$$\bar{W}^* = \frac{\bar{W}}{Z} + \lambda^*|(\bar{K}^*_{xx_{cr}} - \bar{N}^*)| \tag{6}$$

where

$$\bar{W}^* = \frac{W^*}{2\pi L^3\rho_{sk}(1-\nu^2)^{1/2}}; \quad \bar{K}^*_{xx_{cr}} = \frac{\bar{K}_{xx_{cr}}}{Z^3};$$

$$\bar{N}^* = \frac{12R^4\bar{N}}{\pi^2E(1-\nu^2)^{1/2}L^4}; \quad \lambda^* = \frac{\pi EL\lambda}{24\rho_{sk}R^3};$$

$$\bar{W} = 1 + \frac{1}{1-\nu^2}\left(\frac{E\rho_x}{E_x\rho_{sk}}\bar{\lambda}_{xx} + \frac{E\rho_y}{E_y\rho_{sk}}\bar{\lambda}_{yy}\right) \tag{7}$$

It can be shown that there is no minimum $\bar{W}^*$ with respect to a finite Z. Therefore in the "optimization" phase one may generate charts on the $\bar{\alpha}_x - \bar{\alpha}_y$ space for a given load parameter $\bar{N}^*$ and for each Z, of minimum weight parameter $\bar{W}$ with respect to $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ against general instability. This is accomplished by specifying values of α_x and α_y and obtaining the minimum of $\bar{W}$ along with the minimizing values of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$. To this end, the Simplex method of Nelder and Mead[22] is employed, which proved to be extremely accurate and fast, for finding $\bar{W}_{opt}$ in the space of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$. This search technique is designed to adapt itself to the topography of the objective function, elongating down long inclined planes, changing directions in curving valleys, and contracting in the neighborhood of a minimum. The well known method of golden section has been used in combination with the Simplex method in calculating $\bar{K}_{xx_{cr}}$ for each movement of the weight function in the $\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$-space. No convergence problems were encountered in this approach. All of these steps are incorporated in a single computer program.[23]

It has been shown in Ref. 24 that, provided λ^* is sufficiently large, the solution of the minimization of $\bar{W}^*$, Eq. (6), will approach the solution of the minimization of $\bar{W}$ subject to the constraint $\bar{K}^*_{xx_{cr}} = \bar{N}^*$. This implies that, if one uses the optimum weight parameter $\bar{W}$, one will find in the space of $\bar{\alpha}_x - \bar{\alpha}_y$ families of curves of constant optimum $\bar{W}$ and the corresponding optimizing values of the $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ which will be employed in the "design" phase to arrive at the minimum weight geometry, satisfying all constraints.

Design Phase

Assuming that the stresses in the stringers and rings are in uniaxial state and the stresses in the skin are in biaxial state, the stresses in the skin, stringers, and rings before buckling are given by [similar to those in Ref. 12]

$$\sigma_{xxsk} = \frac{-(1+\bar{\lambda}_{yy}-\nu^2)\bar{N}}{h[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]}$$

$$\sigma_{yysk} = \frac{-\nu\bar{\lambda}_{yy}\bar{N}}{h[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]}$$

$$\sigma_{xxst} = \frac{-E_x(1-\nu^2)(1+\bar{\lambda}_{yy})\bar{N}}{Eh[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]}$$

$$\sigma_{yyr} = \frac{\nu E_y(1-\nu^2)\bar{N}}{Eh[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2]} \tag{8}$$

For closely spaced stringers, the local skin buckling and the stringer buckling are both governed by the equations of a flat plate.

In the local buckling analysis, it is assumed that all edges of stiffeners and skin connected to any part of the cylinder are simply-supported. Hence from Ref. 25 one may write

$$\sigma_{xxsk_{cr}} = \frac{\pi^2E}{3(1-\nu^2)}\left(\frac{h}{l_x}\right)^2$$

$$\sigma_{xxst_{cr}} = \frac{\pi^2E_x}{12(1-\nu^2)}\left(\frac{t_x}{d_x}\right)^2\left[\left(\frac{d_x}{l_y}\right)^2 + 0.425\right] \tag{9}$$

Considering only absolute values, the critical stresses of the local buckling of the skin and stringers given by Eqs. (9) must be greater than those given by Eqs. (8) accordingly. Furthermore, the applied stresses must be less than a certain appropriate stress level. Of all ring spacings l_y, obtained from the constraint of stringer buckling, one must select the one (there are many) which does not yield panel buckling. Since the rings are placed on the

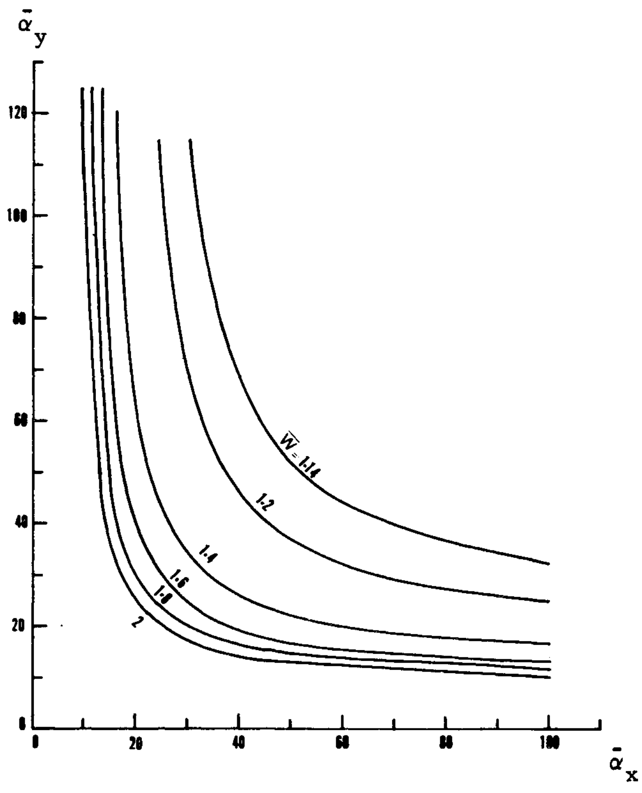

Fig. 2 Design chart for optimum $\bar{W}$. RSRR. $Z = 30{,}000$; $\bar{N}^* = 1.233 \times 10^{-8}$.

inside they are in tension, and therefore there is no possibility of ring buckling.

In the "design" phase the following quantities are known: 1) The applied uniform axial compressive load; 2) The radius and length of the shell; 3) The skin and stiffener materials and their associated properties; 4) The position of the stiffeners (inside). The design variables to be determined are the skin thickness, the ring and stringer sizes and spacings.

The minimum weight design is accomplished in the following manner: 1) For each Z (see step 8) locate the minimum weight parameter $\bar{W}$ in the $\bar{\alpha}_x$, $\bar{\alpha}_y$-space. For example start at any point of the curve labeled $\bar{W} = 1.14$ of Fig. 2. Since the expression for the stress in the rings is based on thin ring theory R/d_y must be greater than 20. This implies that $\bar{\alpha}_y \leqq R/20h$. One then follows steps 2–7 such that no constraints are violated. If any constraint is violated one must move up and down the curve. With 2 or 3 designs it is clear whether or not an acceptable design can be found on that curve. In addition, the designer receives some clear hints for future designs. Finally, if no acceptable design can be found on a single curve, one must increase the weight (move to a new curve or point of higher weight) and repeat the procedure.

2) Calculate the stress levels in the skin, stringer, and rings by employing Eqs. (8). If all stresses are less than or equal to the yield stress or certain limiting stress level, the next step is executed. Otherwise, one must move to the next higher $\bar{W}$ and repeat step 2. Note that since the skin is in a biaxial state of stress one may use the Tresca criterion.

3) The stringer and ring heights are then computed through

$$d_x = h\bar{\alpha}_x; \quad d_y = h\bar{\alpha}_y$$

Note that knowledge of Z implies knowledge of h and the position of $\bar{W}_{\text{opt}}$ in the $\bar{\alpha}_x$, $\bar{\alpha}_y$-space implies knowledge of $\bar{\alpha}_x$ and $\bar{\alpha}_y$.

4) The ratios of the stiffener thickness to the stiffener spacing are determined from the definition of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$, or

$$\frac{t_x}{l_x} = \frac{E\bar{\lambda}_{xx} h}{E_x d_x(1-\nu^2)}; \quad \frac{t_y}{l_y} = \frac{E\bar{\lambda}_{yy} h}{E_y d_y(1-\nu^2)}$$

5) The stringer spacing is determined by requiring that the stress in the skin be less than the skin buckling stress,

$$l_x < h\frac{\pi^2 E}{[3(1-\nu^2)\sigma_{xxsk}]^{1/2}}$$

6) The ring spacing is determined by requiring that the stringer stress be less than the stringer buckling stress.

$$l_y < \frac{d_x}{\left(\dfrac{12(1-\nu^2)}{\pi^2 E_x}\left(\dfrac{d_x}{t_x}\right)^2 \sigma_{xxst} - 0.425\right)^{1/2}}$$

If the square root of the quantity is negative, then any l_y will satisfy the constraint,

$$|\sigma_{xxst_{cr}}| > |\sigma_{xxst}|$$

In this step, l_y must be checked to insure that no panel instability occurs. This is done by a simple computer program through which the panel buckling load is computed and compared to the applied load. Furthermore, the number of rings must be greater than three, for the smeared technique employed to apply.[26] Observe that the failure mode interaction among the general instability, panel instability and local instabilities of skin and stringer can be avoided by proper choice of l_x and l_y.

7) The weight of the stiffened cylindrical shell is then computed by

$$W = 2\pi RLh\rho_{sk}\bar{W}$$

or

$$W = 2\pi RLh\rho_{sk}\left[1 + \frac{1}{1-\nu^2}\left(\frac{E\rho_x}{E_x\rho_{sk}}\bar{\lambda}_{xx} + \frac{E\rho_y}{E_y\rho_{sk}}\bar{\lambda}_{yy}\right)\right]$$

8) Repeat the previous 7 steps for a number of Z values (h) and plot W vs h. It is well known that the skin thickness of the unstiffened geometry is related to the critical load by $h_u = (\bar{N}_{cr}R/0.61E)^{1/2}$. If $\bar{N}_{cr}$ is taken to be the applied load per inch, and since the weight of the unstiffened geometry is greater than that of the stiffened geometry, h_u provides a lower bound for the value of Z. It is anticipated that the optimum stiffened geometry has a skin thickness not less than $h_u/6$. Thus the Z value that corresponds to an optimum $\bar{W}$ lies between Z_u and $6Z_u$. As an initial guess, take $Z = 4Z_u$, generate some data and design the stiffened cylinder according to the previous procedure. Call this weight W_4. Repeat the procedure for $Z = 5Z_u$ and obtain W_5. If $W_4 < W_5$ use $Z = 3Z_u$. If $W_4 > W_5$ use $Z = 6Z_u$. If $W_4 \simeq W_5$ then the minimum weight configuration corresponds to a Z value between $4Z_u$ and $5Z_u$. At least three values of h (or Z) are needed. From the plot of W vs h (or Z) one can locate the absolute minimum weight with the corresponding value of h and hence Z.

9) With the value of Z for minimum weight in step 8 one then generates the required data (design charts) and repeats steps 1–7 to finalize the dimensions. This last step is performed when the exact minimum weight configuration is desired.

IV. Solution Method and Results

The procedure, outlined in Sec. III, is demonstrated through two examples labeled Case 1 and Case 2. The cylinder geometry and load condition for these two design examples taken from Jones and Hague[16] are

Case 1: $R = 95.5$ in., $L = 291$ in., $\bar{N} = 800$ lb/in.
$E = E_x = E_y = 10.5 \times 10^6$ psi
$\rho_{sk} = \rho_x = \rho_y = 0.101$ lb/in.^{3}
$\nu = 0.33$, $\sigma_y = 50{,}000$ psi
$\bar{N}^* = 1.233 \times 10^{-8}$

Case 2: $R = 9.55$ in., $L = 38$ in., $\bar{N} = 800$ lb/in.
$E = E_x = E_y = 10.5 \times 10^6$ psi
$\rho_{sk} = \rho_x = \rho_y = 0.101$ lb/in.^{3}
$\nu = 0.33$, $\sigma_y = 50{,}000$ psi
$\bar{N}^* = 4.10307 \times 10^{-8}$

The results of the minimization of the weight parameter are shown in Figs. 2–4. For Case 2, where $\bar{N}^* = 4.10306 \times 10^{-8}$,

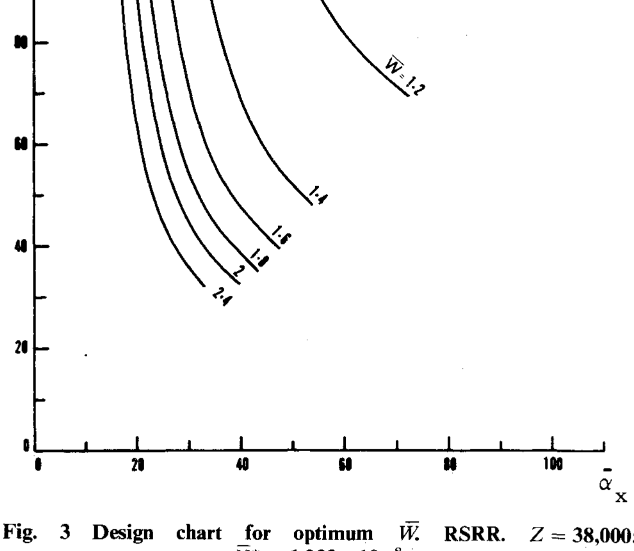

Fig. 3 Design chart for optimum $\bar{W}$. RSRR. $Z = 38{,}000$; $\bar{N}^* = 1.233 \times 10^{-8}$.

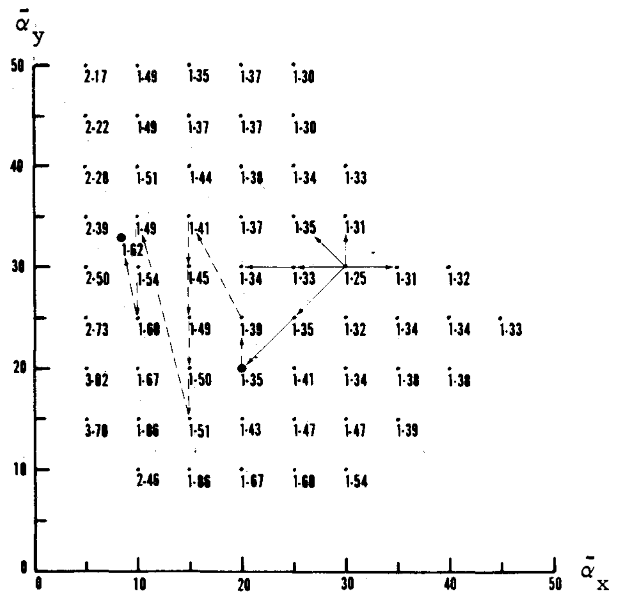

Fig. 4 Design chart for optimum $\bar{W}$. RSRR. $Z = 12{,}000$; $\bar{N}^* = 4.10306 \times 10^{-8}$.

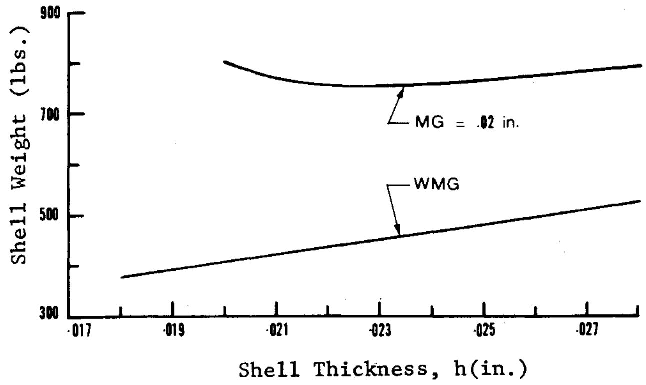

Fig. 5 Minimum stiffened shell weight vs shell thickness. Case 1.

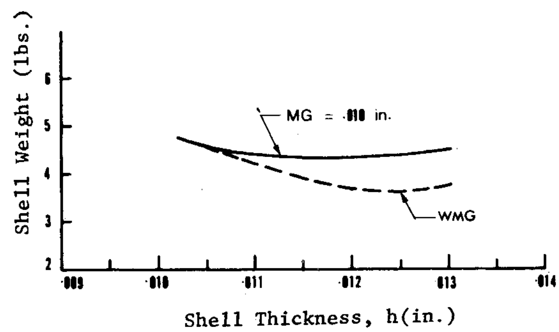

Fig. 6 Minimum stiffened shell weight vs shell thickness. Case 2.

the surface of optimum $\bar{W}$ becomes wavy, hence smooth curves cannot be drawn, therefore only 1 chart is presented in Fig. 4 as an example for designing purposes. Together with the charts, the values of $\bar{W}$, $\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$, m, and β for each $(\bar{\alpha}_x, \bar{\alpha}_y)$ are also tabulated.[23] These tables are not included in this paper in order to save space.

In Fig. 4, the solid lines are the movement in designing Case 2 without geometric constraint while the dashed lines show how the path of the design is taken to satisfy the geometric constraint that no dimensions will be less than 0.01 in. The small circles indicate the successful designs.

The minimum gage as used in these examples implies that no dimension (skin thickness, stringer, and ring thickness) are smaller than the specified value. The methodology proposed is general and the geometric constraints (MG) could be applied to some or all thicknesses.

The results of the design analysis are shown in Figs. 5 and 6. Observe that for the lightly loaded shell where yielding is not a strong factor the plot of W vs h for the design WMG (without minimum gage) is a straight line as in Fig. 5. For Case 2, the heavily loaded shell, where yielding is critical the curve of W vs h concaves downward. The final designs for minimum weight are shown in Table 1. Comparisons to the design results of Jones and Hague are also shown in the table but their designs have the interaction among the buckling modes of failure. This is seen through the lower part of the table in which the different failure loads are compared to the applied load (see Nomenclature). These numbers should not exceed *one*.

In Case 1, the weight improvement is 45.3% for the design WMG. There is no weight improvement for Case 2, WMG.

Table 1 Design results and comparison

	Case 1			Case 2		
	WMG (present)	WMG (Ref. 16)	MG = 0.02 in.	WMG (present)	WMG (Ref. 16)	MG = 0.01 in.
W	373	682.54	755	3.707	3.700	4.360
h	0.018000	0.03044	0.022105	0.011895	0.00998	0.010980
t_x	0.000527	0.02760	0.032620	0.004424	0.01244	0.014921
t_y	0.000004	0.000022	0.022720	0.000235	0.00027	0.014937
d_x	2.07000	0.3879	0.44210	0.23789	0.11348	0.09882
d_y	2.07000	20.0000	2.10000	0.23789	1.00850	0.32939
l_x	0.51970	1.3162	0.91985	0.32072	0.23791	0.29114
l_y	0.00800	3.2290	9.38710	0.05994	1.65190	1.18750
GB	1.0000	1.0028	1.0000	1.0000	1.0042	1.0000
PB	0.0003	0.2173	0.9017	0.0006	0.9943	0.7339
SB	0.8511	1.0051	0.9542	0.9029	0.7486	0.9198
STB	0.9427	1.0071	0.9292	0.8879	1.0007	0.5159
SY	0.7964	0.4145	0.4269	0.9687	1.0030	1.0130
$STYC$	0.7925	0.4146	0.4186	0.9620	1.0039	0.9893
RYT	0.2487	0.1375	0.1146	0.2966	0.3295	0.2430
m	8	27	18	7	13	16
n	10	6	9	8	7	7
m_p	1	1	1	1	1	1
n_p	over 600	62	36	272	21	25

Note that in Case 1, WMG the weight can be further reduced since skin yielding is only 0.7964. This will require additional data where h is less than 0.018 in. It is interesting to note that the design steps 4, 5, and 6 yield many combinations of t_x, t_y, l_y which lead to the same weight.

V. Conclusions

The important conclusions of the present investigation are: 1) The solution of the problem is not unique. This means that there are many combinations of the design variables for the same minimum weight. 2) The approach allows the designer to deviate from the optimum solution, with minimum-weight penalty, to avoid failure mode interaction and/or unrealistic stiffener geometries. 3) The generated data can be used for other circular cylindrical shells and loading whose $\bar{N}^*$ is about the same. If the generated charts are stored eventually all the possible cases of $\bar{Z}$ and $\bar{N}^*$ will be covered and there will be no need to generate more charts but simply use the stored ones in the "design" phase.

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